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Special Contribution: Beyond the Swimming Propulsive Force: Bridging Conventional Biomechanics and the Energy–Momentum Continuum

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ABSTRACT

BACKGROUND: Swimming biomechanics still explains propulsion as the hands and feet 'pushing water backwards'. However, years of flow visualisation have revealed a different story. During a cycle, the hand does not propel the body by pushing a compact mass of water. Instead, it continuously modifies the surrounding flow, creating pressure gradients, fluid accelerations and vortical structures. These evolving flow structures redistribute momentum and mechanical energy throughout the surrounding water. Consequently, the hydrodynamic force measured at a particular instant only represents one aspect of the interaction. A complete physical description must also account for the transient storage, convective transport, and dissipation of mechanical energy within the flow field generated by the swimmer. We therefore describe human aquatic propulsion as an energy–momentum continuum, and argue that the force-based and continuum concepts are complementary rather than competing. **METHODS** In a conceptual, conference-length format, the case is built in three linked steps — “Bridging the Gap”, “Decoding the Mechanics of Speed”, and “Beyond Force” — explained in an illustration in each case. The reasoning rests on the Navier–Stokes equations, on global energy and momentum balances, on a Helmholtz decomposition of the flow, and on a BIV–CFD validation chain; derivations are kept to a minimum. **RESULTS:** The framework redefines the single force value per cycle in terms of power flow (in watts), energy turnover per cycle, momentum transfer and work cycles. It explains why the Strouhal number determines which mechanism dominates in each body segment, yielding quantities that can be used by coaches and equipment designers. **CONCLUSIONS:** "Beyond force" means thinking past it by putting energy, momentum and vortices at the centre, while keeping what kinematics has already taught us.

KEYWORDS: swimming biomechanics | aquatic locomotion | energy–momentum continuum | hydrodynamics | vortex dynamics

INTRODUCTION

Two ways of describing how the body moves through water are usually treated as rivals: the conventional mechanical view of body movement, and the continuum-oriented perspective that builds on the exchange of energy and momentum at the body–water boundary. We do not think they are rivals. They describe the same events at different levels, and the field gains nothing by forcing a choice between them.

Decades of measurement have catalogued the kinematics in fine detail: the foot actions and their heaving and pitching in the leg kick, and the sculling hand actions with their changing orientation to the water. The flow itself has fared worse: the ring-shaped structures left in the water are rarely reported, and we still lack a fully integrated, experimentally validated theory linking whole-body swimmer kinematics, three-dimensional wake dynamics, metabolic power and forward speed. That gap — between what the body does and what the water does — demands further research. Biologists studying fish and marine mammals reached this point years ago: for them, unsteady, vortex-based propulsion, not a quasi-steady balance of forces, is the standard picture[1,2,3]. Our aim is to apply this approach to human swimming.

One observation drives the whole re-framing, and we want to put it plainly, because it contradicts the terminology the field uses every day. Watching the flow for many years, we keep seeing the same thing: the water around a working hand does not move backwards: in the laboratory frame it stays more or less where it is; the hand moves through and around it and the fluid wraps about the hand. Velocimetry makes the point sharper. At the palm — the face that meets the oncoming water — the flow relative to the moving hand decelerates to near zero, forming a local stagnation region where the pressure rises — and this elevated pressure is itself part of the propulsive effect. The water moves merely on the dorsum of the hand, where the flow separates and rolls up into vortices and a turbulent wake per cycle. Particle-

image velocimetry of the swimming hand shows the same strongly unsteady field, with a pair of counter-rotating vortices[4]. The propulsive reaction, then, is the organisation of unsteady flow — pressure gradients, virtual added-mass effects, circulation, separated shear layers, vortices and wake momentum — rather than the backward displacement of a coherent volume of water; whenever this paper refers to 'pushing water backward,' it should be interpreted exactly that way.

This paper presents a comparison between the classical force-based description and the energy–momentum description, demonstrating their intersection and offering terminology and a method that transforms energetic quantities into actionable insights for coaches and designers. We clearly demonstrate that power flow, energy turnover and momentum transfer provide a more useful insight into swimming than a series of instantaneous force values. Given the conference paper format, we focus on the concepts and defer the equations -except one- and derivations to the supplementary material.

METHODS

Conceptual approach

We set out our view of self-generated swimming in three steps, borrowing the visual logic of a graphic novel. They are “Bridging the Gap” (an overview), “Decoding the Mechanics of Speed: A Unified Vision for Human Aquatic Propulsion” (which goes further), and “Beyond Force: The Energy-Momentum Continuum in Swimming Biomechanics” (the full statement of the view). Each is explained in the text below and pulled together in one figure (Figure 1; the prompt used to generate it is provided as a separate prompt sheet). More detailed treatments, including equations, are available elsewhere, including on ResearchGate.

Analytical framework

The supporting analysis combines four elements: (i) decomposition of local force fields and their linkage to global energy and momentum balances, in order to localise interactions and quantify energy storage and waste; (ii) Helmholtz projection, which splits the velocity field into a potential (displacement / virtual added-mass) component and a vortical (vortex / wake) component, complemented by modal and Koopman analyses of the vortical field; (iii) compliance models, implemented through finite-element contact chains, to represent deformable surfaces within fluid–structure interaction; and (iv) assessment of cyclic effects through time-resolved momentum and energy fluxes. Energy and momentum integrals smooth local measurement errors and provide a consistent validation ladder between experiment — Bubble Image Velocimetry (BIV) — and computational fluid dynamics (CFD). A fundamental law governs every one of these elements: The motion of the water is described by the incompressible Navier–Stokes equations, which express the conservation of momentum for a viscous fluid:

$$\rho(\partial u/\partial t + (u\cdot\nabla)u) = -\nabla p + \mu\nabla^2 u + f \qquad (1)$$

where u is the velocity field, p the pressure, ρ the density, μ the dynamic viscosity, and f a body force. Taken term by term, it shows why the energy–momentum view is forced on us by the physics, not chosen for convenience. The unsteady term $\rho\,\partial u/\partial t$ embodies virtual added-mass and history effects — the water’s “memory”. The nonlinear convective term $\rho(u\cdot\nabla)u$ is the origin of vortex formation and turbulence, and it is precisely this nonlinearity that prevents the flow from being separated a priori (that is, from first principles, before any appeal to the observed flow) into independent “propulsive” and “braking” contributions. The pressure-gradient term $-\nabla p$ is the field that, at the hand, produces stagnation at the palm and suction behind the dorsum. The viscous term $\mu\nabla^2 u$ governs the boundary layer, frictional dissipation, and the basis for compliant-surface drag control. Because no closed analytical solution exists for the unsteady, high-Reynolds-number flow around a swimmer, integral energy and momentum balances — formally, the Navier–Stokes equations integrated over a control volume — together with the Helmholtz projection of the velocity field, become the descriptors that actually work[2].

Two aspects of Equation (1) are essential for the framework proposed here. First, water particles are continuously accelerated not only because the flow changes with time but also because they move through an evolving flow field. Swimming therefore involves both local and convective acceleration throughout the stroke cycle. Second, the flow around a swimmer is too complex to describe by solving every detail directly. Instead, fluid mechanics considers the overall balance of momentum and energy within a control volume surrounding the swimmer. From this perspective, propulsion is understood as the continuous transfer of momentum and mechanical energy into the wake rather than as the result of isolated instantaneous forces. Because competitive swimming occurs at very high Reynolds numbers ($Re \approx 10^6$), the flow is dominated by convection, vortex formation and unsteady three-dimensional structures, making a purely quasi-steady force description physically incomplete.

One common misconception should be addressed at the outset. Under unsteady flow conditions, the decomposition of the resultant hydrodynamic force into separate "propulsive" and "braking" components is not uniquely defined and cannot be measured independently in the same way as under steady-flow conditions. This limitation motivates the shift from local force components to integral descriptors based on momentum and mechanical energy, which provide a more consistent description of swimmer–fluid interaction.

RESULTS

The framework is set out below through the three perspectives, then through its scientific and technical payoff. Figure 1 summarises the whole.

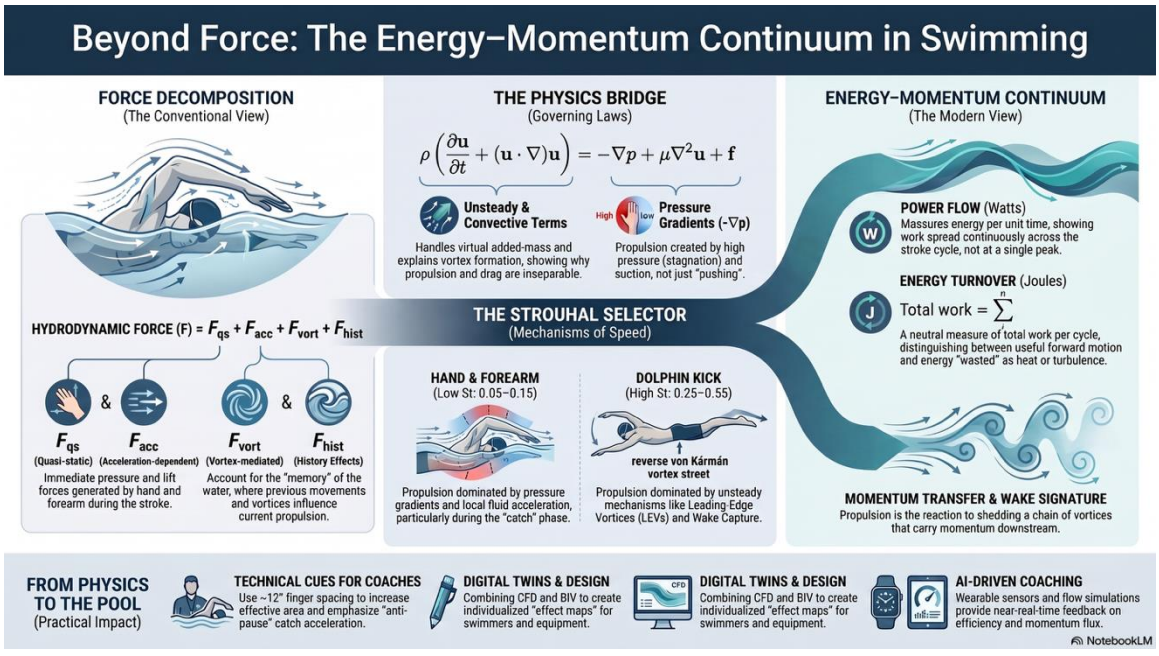


Figure 1. Conceptual synthesis of the energy–momentum continuum in swimming. Figure created with Gemini NoteBook LM.

Bridging the Gap: from force to energy transformation

Conventional swimming biomechanics explains propulsion in terms of forces: the hands and feet "push water backwards" and the water reacts with a counter-force. As noted above, that sentence needs care. During a cycle, the hand does not propel the body by pushing a compact mass of water. Instead, it continuously modifies the surrounding flow to create pressure gradients, fluid accelerations and vortical structures. These evolving flow structures redistribute momentum and mechanical energy within self-induced flow. Consequently, the hydrodynamic force measured at a particular instant only

represents one aspect of the interaction. A complete physical description must also account for the transient storage, convective transport and dissipation of mechanical energy within the flow field generated by the swimmer.

With that correction in place, the fluid dynamics gets more interesting, not less. Moving body parts do not merely accelerate water; they also generate vortices and displace water, creating further hydrodynamic effects. There is also a historic reminiscence: what the hand did a moment ago still shapes the flow it meets now, so the water has a short memory. Propulsion is therefore not one force but the combined result of pressure gradients, acceleration, vortices, and left-over flow.

"Bridging the Gap" is the response to this. It joins the conventional analysis of forces with an examination of energy and momentum flows. A force tells you about a single instant; an energy account tells you where the energy supply actually went. Not every transformation does useful work: only part of the energy results as forward motion, while the rest ends up as waste — heat, turbulence, and other irrecoverable forms. Vortices demonstrate this clearly: a force trace only records their effect, whereas an energy analysis provides a more complete picture by revealing whether the generated vortices enhance locomotion by storing and transferring mechanical energy, or contribute to turbulent dissipation. Two swimmers can achieve the same speed yet incur very different energetic cost[s]. The chain of events begins with the muscular action, continues with the water that is set in motion, and ends with the body's own acceleration.

The Strouhal number ties together frequency, amplitude, and swimming speed, and helps identify the likely propulsive regime and wake organisation, though it does not by itself determine the mechanism, which also depends on Reynolds number, morphology, flexibility and three-dimensional kinematics. For the hand in front crawl, it is low, and pressure gradients, drag, and acceleration — especially at the catch — dominate. The dolphin kick, Strouhal number runs higher, and their vortices matter: under favourable foot orientation and phase timing, leading-edge-like vortices may contribute to thrust[7], and swimmers can recover vortices shed earlier in the cycle (wake capture), in a manner analogous to the wake capture described in animals and foils, although direct evidence in human swimmers remains limited[1]. Fast swimming animals achieve their highest efficiency at Strouhal numbers between about 0.2 and 0.4 at cruising speed[8]. Human underwater undulation, however, usually imply somewhat higher Strouhal numbers when combining motion of the trunk, pelvis and legs resulting in whip-lash like action to tips of the feet. Recent particle-image-velocimetry studies of human underwater undulatory swimming show that peak vorticity and the orientation of the inter-vortex jet vary with swimming speed[9]. A swimmer is surrounded by a single, continuous flow field. Every visualization experiment, from dye to CFD simulation confirms that water behaves as a continuum. This simple observation suggests that propulsion should also be understood within a continuous framework of energy and momentum transport rather than as a collection of isolated force events.

From this perspective, mechanical power is a more informative measure than peak force alone. The ability to swim fast depends not only on producing large forces, but also on transferring energy to the water efficiently throughout the stroke cycle. While force describes the strength of an interaction at a given instant, power describes the rate at which energy is transferred. CFD, together with BIV measurements, lets us look at the pressure field, the vortices, and the energy flow, and check the models against what swimmers actually do. Sensors, data analysis, and machine learning increasingly make it possible to assess the energy efficiency of a stroke directly. A few practical points follow: slightly spread fingers can improve flow organization, a well-controlled catch may promote the formation and stabilization of beneficial leading-edge vortices (LEVs), and a precisely timed dolphin kick can generate more coherent vortex structures that enhance momentum transfer.

Decoding the mechanics of speed: a unified perspective

Why are some swimmers faster than others — muscle and technique, or something else? For a long time, propulsion was seen mainly as a result of the force of the hands and feet on the water. Our own flow-visualization studies suggest that the classical picture of the hand pushing a compact block of water backwards is incomplete. Instead, the hand

appears to continuously reorganize the surrounding flow, generating pressure gradients, local fluid accelerations and coherent vortical structures that redistribute momentum throughout the swimmer-generated flow field. The reality is more complex: the self-induced flow surrounding continuously the swimmer reorganizes into pressure gradients, accelerated water masses and vortical structures whose evolution depends on the history of the flow. The instantaneous hydrodynamic force is therefore only one manifestation of this coupled process, while propulsion emerges from the combined transfer of momentum and mechanical energy.

The Strouhal number as a key shows: pressure and acceleration predominate arm/hand effects and vortices induce locomotion due to dolphin-like kicking. This reactive propulsion mechanism—thrust generated through the transfer of momentum to the surrounding fluid—provides a common physical framework linking Lighthill's elongated-body theory[10] of undulatory fish locomotion with the oscillating-hydrofoil model[3] of cetacean flukes. Although the kinematics differ, both mechanisms rely on unsteady fluid acceleration, momentum transfer and vortex-mediated flow structures to generate efficient propulsion. Different segments play different roles: the hand and forearm act mainly through pressure and acceleration, the feet shed vortices, while the head and trunk mainly affect the swimmer's resistance.

Seen through this lens, the modern account of animal swimming carries over to the human case with few changes. An oscillating propulsor does not so much push against the water as roll it into vortices and cast the resulting momentum downstream: each shed vortex carries a hydrodynamic impulse, and by Newton's third law the reaction to that impulse is thrust. In steady swimming these vortices organise into a reverse von Kármán street — a chain of counter-rotating cores with a jet threading between them — whose rearward momentum is the signature of a propulsive, rather than a braking, wake[2]. How much impulse a single event can deliver is itself bounded: a starting vortex grows only until it pinches off, so there is an optimal vortex-formation number beyond which further fluid is shed as a trailing jet rather than added to the ring, a principle that unifies pulsed jetting, fish tails and flapping fins[11]. A propulsor timed to meet and re-ingest vortices it has already shed — wake capture — can recover part of that energy, raising efficiency at no extra muscular cost. These are not merely biological analogies: the counter-rotating vortex pair behind the swimming hand[4], the speed-dependent foot vortices of underwater undulatory swimming[9], and the bubble- and particle-based visualisations of the propulsive wake reported by the present authors[12,13] all reveal the same unsteady, vortex-dominated field in humans. What the energy–momentum framework adds is a way to quantify it: the impulse of each vortex, the fraction of invested work that leaves as recoverable wake energy, and the Strouhal- and Reynolds-dependence of when each mechanism is worth using.

Two approaches are used together: computer models simulate the flow around the swimmer, while in-pool measurements capture how the water moves. Comparing the two shows whether the models are right and closes the loop between theory and the pool. Even small details matter: a slight finger speading (about 12 degrees)[14] increases the effective hydrodynamic area of the hand and raises its drag coefficient by roughly 9%, an effect that depends on hand orientation, speed and the chosen force criterion, while the catch, regarded as the initiation phase of flow induction, require a finite development time to progressively establishes the pressure field and vortical structures that subsequently mediate efficient momentum and energy transfer.

This paper is moving from generic models to analyses tailored to the individual swimmer. Propulsion is generated not by the hand alone but through the coordinated arm action, e.g. the orientation and motion of the forearm strongly influence the flow developing around the hand, illustrating the hydrodynamic coupling between adjacent segments. With 3D scanning, one could build individualised "effect maps" for each swimmer. The roadmap has four strands: whole-body hydrodynamics, which treats the swimmer as a whole — including starts and turns, where wave resistance and full-body motion matter most; equipment design, where simulation and AI fit paddles or fins to the individual and estimate propulsive forces in real time, guided by scaling laws for heaving and pitching foils[15] and helped by machine-learning methods now common in fluid mechanics[16]; individual anatomy, where foot size, body proportions, and ankle mobility affect flow and efficiency; and a digital ecosystem — in effect a digital twin of the swimmer — where multi-camera footage

feeds flow simulations while wearable sensors feed AI models during training, so that a coach can get near-real-time feedback on how well a stroke or kick is working.

Beyond Force: the energy–momentum continuum

"Beyond Force" changes what we look at. Instead of reading off isolated force values part by part, we follow the continuous flow of energy — from swimmer to water — and the matching changes in momentum. The limits of the force picture are easy to see. Propulsion is usually treated as a single force averaged over a whole cycle, which holds only at constant speed, and a swimmer is rarely or never at constant speed. Water is not a rigid body but a moving medium full of vortices, turbulence, and interaction; a single number cannot capture that. Nor can a force be split cleanly into "propulsive" and "braking" parts, because the moving body keeps making new flow that does several things at once. So a force-only account misses vortex structures, turbulent wakes, and energy stored in the water — because an isolated instantaneous force value does not, by itself, quantify the impulse, work, wake kinetic energy, recoverable vortex energy or irreversible dissipation that determine performance. Our flow and velocimetry results show this directly: with the water at rest at the palm and moving only behind the dorsum, what matters for propulsion is the momentum and energy carried by the wake, which a single force number cannot represent.

At the centre of this view is power flow, measured in watts, together with energy turnover (work) per cycle, measured in joules per stroke cycle and a common currency for biological and physical work alike. What counts is not the peak force but the efficiency of the flow; energy and momentum based on mass (kg), length (m), and time (s). The swimmer sits inside the flow: energy bands, streamlines, and vortices run through the body — a reminder that propulsion comes not from the body alone but from the exchange between the body and the water it is shaping and using.

The vocabulary is deliberate, and it goes back to the conservation of energy and momentum — the same laws the modern hydrodynamics of swimming is built on[2]. Four terms describe cyclic propulsion better than "force": Power-induced Flow — power as energy per unit time, with "flow" marking the continuous spread of watts across the cycle; Energy Turnover per Cycle — a neutral, exact measure of how much energy is put in or given back per stroke; Momentum Transfer — the redistribution of fluid momentum within and across a control volume — pressure forces, viscous stresses and wake momentum flux — that produces the hydrodynamic reaction on the swimmer's mass, the efficiency of which depends on how vortices are formed and shed[11]; and Work Cycle — work as energy, with "cycle" marking the repeated cycle of energy absorption, transfer, and dissipation. The point is not novelty but clarity: to describe cyclic energy transfer without collapsing it into single, isolated numbers.

Scientific and technical benefits

This framework combines local force decomposition with global energy balances. It identifies where hydrodynamic interactions occur within the unsteady flow while quantifying how mechanical energy is transferred, temporarily stored and ultimately dissipated. The main benefits follow.

- Physical completeness. Linking decomposed forces to global energy balances reveals not only where interactions occur within a continuous flow field, but also how energy is distributed, stored, or wasted — which is especially important for convective flows and for assessing cyclic energy recovery. Small changes in contour, or a compliant surface, can smooth the sharp pressure gradients that would otherwise trigger flow separation.
- Robust data validation. Energy and momentum integrals smooth local measurement errors (for example, BIV near-wall artefacts) and enable more consistent CFD↔BIV comparisons across scales. Key metrics include local kinetic energy, the dissipation rate ε, the proportion of vortical energy, the time-integrated work per cycle, and the cycle-integrated recovery rate.
- Disentangling mechanisms. Helmholtz decomposition separates the unsteady field into potential (displacement / virtual added-mass) and vortical (vortex / wake) components, supporting a mechanistic interpretation of displacement, virtual added-mass and wake-related energy pathways; causal attribution, however, requires

explicit boundary conditions and validation against experiment or CFD. Cyclic motions re-energise the boundary layer and can suppress flow separation (stall).

- Training cues. Energy-flow diagnostics give concrete information on phase timing, acceleration profiles, and required joint mobility, from which immediately actionable technique cues and direct performance metrics (efficiency η, lap and race times) can be derived.
- Design guidance. The framework translates energy-flow analyses into design specifications for equipment (for example, paddle profile and fin stiffness), establishing energy recovery and cyclic waste as measurable design objectives.

Applied to prototypical paddle and fin shapes, the framework enables the quantitative separation of energy pathways (displacement versus vortices) over a single stroke cycle, the validation of CFD models against BIV data using integral metrics, the identification of design-relevant parameters for increasing recovery and reducing cyclic waste, and the derivation of training interventions that optimise phase timing and acceleration profiles.

DISCUSSION

The two descriptions are complementary, not rival. The conventional mechanical motion is still there to see; but its effect appears only in how the surrounding water responds to it. The energy–momentum view does not replace the established kinematic work; it extends it, and puts it on consistent physical footing at the body–water boundary. A good example, partly unpublished, makes the point: the hand–forearm can produce locomotion without a push/pull backward, driving the swimmer forward while leaving a propulsive wake on a Z-shaped path. Body kinematics and energy transfer are fundamentally coupled, requiring a unified rather than separate interpretation. The analogy with animals is more than a metaphor: early studies already linked the human butterfly and dolphin-kick to the symmetric fluke beat of cetaceans[17], and across many fishes steady swimming settles on the same shape of body wave[18], which points to strong hydrodynamic constraints; human underwater undulatory swimming may approach some of these functionally, although anatomical and task-specific limitations prevent direct equivalence. Energy-flow measures, in particular, show when invested work is stored as recoverable vortex energy and when it is irrecoverably wasted. The same thinking lights up the old puzzle of cetacean drag, Gray's paradox, which modern work now regards as largely dissolved through revised estimates of muscle power and drag, with unsteady, vortex-based propulsion and passive or active flow control as secondary contributors rather than the core resolution[19,3], both with direct counterparts in swimwear and stroke technique. A further distinction deserves emphasis: the virtual added mass — the inertial reaction of the fluid accelerated with the hand, in the sense taught by Lighthill and Webb — should not be confused with the additional mass of water that a swimmer sets surging, phase-shifted by roughly 200 ms, and that can be seen running up the wall at the end of a race. This phase shift between body action and fluid response is itself a variable worth tracking, and it is where the coupled, multi-scale description of the swimmer–water system becomes essential.

Limitations

This document presents a conceptional framework without new experimental data or formal validation protocols and could be regarded as hypothesis-generating, that requires systematic experimental validation. Concerning Particle Image Velocimetry (PIV), its post-processing and reconstruction procedures may an open methodological question deserving further discussion. Finally, several aspects presented here are based on ongoing, as yet unpublished work and should therefore be regarded as testable hypotheses rather than established conclusions.

Practical Applications

The message to take away: fast swimming comes not from making more force but from trading energy and momentum more effectively between the body and the water. In practice, this means open analysis tools — open-source software, combined CFD–BIV pipelines, Helmholtz projection, and modular modal analysis — that turn energetic quantities into training cues (phase timing, acceleration profile, joint mobility) and design guidance for equipment (paddle profile, fin

stiffness). The aim is to develop a research programme that translates energetic parameters into training and design decisions, thereby closing the gap between observation and physical explanation.

CONCLUSION

'Beyond Force' does not mean replacing force, but recognising that it is merely one manifestation of a much richer physical process. Swimming is fundamentally an exercise in the continuous redistribution of energy and momentum within a fluid continuum. Viewing the swimmer and the surrounding water as a coupled dynamic system creates opportunities for new physical observables, more rigorous validation strategies and evidence-based training principles. After decades of focusing primarily on body kinematics, the field is ready for a broader paradigm that integrates biomechanics, hydrodynamics and physiology into a unified description of human aquatic locomotion.

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